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Duality-Based Joint Optimization Scheme with Low-Complexity for Cooperative MIMO Systems

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ABSTRACT

In this study, a joint beamforming and power optimization scheme using Lagrangian duality was proposed for cooperative multiple-input multiple-output systems. Despite its very low complexity, the proposed method achieved the same performance as the conventional joint optimization method based on an iterative method.

Key Words : MIMO, Beamforming, Joint optimization, Lagrangian duality, Low complexity

Ⅰ. Introduction

Cooperative multiple-input multiple-output (MIMO) is a highly efficient transmission technology for B5G cellular networks. Recently, a joint optimization algorithm was proposed for minimizing the overall transmit power in multicell multiuser MIMO systems [1]. This algorithm is an iterative joint optimization problem (I-JOP) that jointly computes the transmit/receive beamforming vectors and transmits power. Although I-JOP provides an iterative solution for determining the optimal transmit power, its computational complexity must be considered as the number of users and cells increases. Thus, in this study, a new JOP is proposed to compute the minimum transmit power using Lagrangian duality to achieve the same performance while solving the complexity problem of the I-JOP.

Ⅱ. System Model

In this study, we assume the same system model as in [1] to ensure fair performance comparison. Thus, we considered a multi-cell multiuser MIMO system with N cells having N_t transmit antennas and K users with $N_r \geq N_t$ receive antennas in each cell, where $K \leq min\{N_r, N_t\}$. Further details are available in [1], which may be summarized as follows. Let $x_{i,j}$ and $y_{i,j}$ denote the complex signal transmitted to the jth user in the ith cell and the signal received by the jth user in the ith cell, respectively. $\mathbf{H}_{ij}$ represents the $N_r \times N_t$ channel matrix from the BS in the ith cell to the jth user, while $\mathbf{G}_{n,i}$ represents the $N_r \times N_t$ interference signal matrix from the nth cell to the user in the ith cell. Channels $\mathbf{H}$ and $\mathbf{G}$ are assumed to be Rayleigh fading and $n_{i,j}$ indicates zero-mean additive white Gaussian noise with variance σ^2 at the jth user in the ith cell. Additionally, $\mathbf{f}_{i,j}$ and $\mathbf{w}_{i,j}$ denote $N_t \times 1$ normalized precoding and $N_r \times 1$ normalized postcoding vectors, respectively. Only one stream per user is assumed to be transmitted from each BS and each user has its own power constraint ($P_{i,j}$).

Then, the received signal for the jth user in the ith cell can be rewritten as

$$\begin{aligned} y_{i,j} &= \mathbf{w}_{i,j}^{*}\mathbf{H}_{i,j}\mathbf{f}_{i,j}\sqrt{P_{i,j}}x_{i,j} \\ &+ \sum_{k\neq j}\mathbf{w}_{i,j}^{*}\mathbf{H}_{i,j}\mathbf{f}_{i,k}\sqrt{P_{i,k}}x_{i,k} \\ &+ \sum_{n\neq i}\sum_{k}\mathbf{w}_{i,j}^{*}\mathbf{G}_{n,i}\mathbf{f}_{n,k}\sqrt{P_{n,k}}x_{n,k} + \mathbf{w}_{i,j}^{*} \end{aligned} \tag{1}$$

For the given beamforming vectors $\mathbf{f}_{i,j}$ and $\mathbf{w}_{i,j}$, where $j = 1, \cdots, K$ and $\mathrm{i} = 1, \cdots, \mathrm{N}$, the signal to interference plus noise ratio (SINR) for the jth user in the ith cell can be expressed as

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$$\Gamma_{i,j} = \frac{P_{i,j}\left|\mathbf{w}_{i,j}^{*}\mathbf{H}_{i,j}\mathbf{f}_{i,j}\right|^{2}}{\sum_{k\neq j}P_{i,k}\left|\mathbf{w}_{i,j}^{*}\mathbf{H}_{i,j}\mathbf{f}_{i,k}\right|^{2}+\Gamma_{i,j}^{I}} \quad (2)$$

where $\Gamma_{i,j}^{I} = \sum_{n\neq i}\sum_{k}P_{n,k}\left|\mathbf{w}_{i,j}^{*}\mathbf{G}_{n,i}\mathbf{f}_{n,k}\right|^{2}+\left|\mathbf{w}_{i,j}^{*}\right|^{2}\sigma_{i,j}^{2}$.

When $\gamma_{i,j}$ is the SINR target for the jth user in the ith cell, the transmit power minimization problem can be expressed as

$$\begin{aligned}&\text{minimize } \sum_{i}\sum_{j}P_{i,j}\left|\mathbf{w}_{i,j}^{*}\mathbf{H}_{i,j}\mathbf{f}_{i,j}\right|^{2}\\&\text{subject to } \Gamma_{i,j}\geq\gamma_{i,j}\end{aligned} \quad (3)$$

where $i = 1, 2, \cdots, N$, $j = 1, 2, \cdots, K$. As noted in [1], (3) can be considered as an individual optimization problem (IOP).

Ⅲ. Duality-Based Joint Optimization Scheme

In this section, we propose a new JOP to compute the minimum transmit power problem using Lagrangian duality.

The I-JOP proposed in [1] for determining the joint power allocation algorithm at the nth iteration is given by

$$P_{i,j}^{n+1} \approx \frac{\gamma_{i,j}P_{i,j}^{n}}{\Gamma_{i,j}} \quad (4)$$

Although the I-JOP algorithm performs better than the IOP, its computational complexity becomes considerable as the number of users and cells increases. The transmit power update algorithm in equation (4) jointly computes the NK powers $P_{i,j}$, per iteration.

The uplink beamforming and power optimization problems are much easier to solve than those in the downlink case. The downlink capacity of a MIMO broadcast channel can also be computed using the duality of the uplink capacity[2,3]. The original algorithm can be solved efficiently by transforming the downlink problem into the dual-uplink problem. The approach in [4] proposed an uplink-downlink duality using a Lagrangian dual of the optimization problem when only transmit beamformers were used. We expand the problem to include both transmit and receive beamformers to maximize the effective SINR.

Because the SINR constraint is a convex problem and the maximum ratio combining receive beamformer is a function of its transmit beamformer, $\mathbf{w}_{i,j} = \mathbf{H}_{i,j}\mathbf{f}_{i,j}/\left\|\mathbf{H}_{i,j}\mathbf{f}_{i,j}\right\|$, the Lagrangian of (3) can be written as

$$\begin{aligned}L(\mathbf{f}_{i,j},\lambda_{i,j}) = &\sum_{i}\sum_{j}P_{i,j}\left|\mathbf{w}_{i,j}^{*}\mathbf{H}_{i,j}\mathbf{f}_{i,j}\right|^{2}\\&-\sum_{i}\sum_{j}\lambda_{i,j}[A-B-\sigma^{2}]\end{aligned} \quad (5)$$

where $A = P_{i,j}\left|\mathbf{w}_{i,j}^{*}\mathbf{H}_{i,k}\mathbf{f}_{i,k}\right|^{2}$ and $B = \sum_{i}\sum_{j}A - A$. Because the dual objective is given by $g(\lambda_{i,j}) = min_{\mathbf{f}_{i,j},P_{i,j}}L(\mathbf{f}_{i,j},\lambda_{i,j})$, the Lagrangian dual of the original JOP problem can be expressed as

$$\begin{aligned}&\text{maximize } \sum_{i}\sum_{j}\lambda_{i,j}\sigma_{i,j}^{2}\\&\text{subject to } P_{i,j}+\sum_{i}\sum_{j}\lambda_{i,j}\mathbf{H}_{i,j}^{*}\mathbf{H}_{i,j}\\&\qquad\geq\left(1+\frac{1}{\gamma_{i,j}}\right)\lambda_{i,j}\mathbf{H}_{i,j}^{*}\mathbf{H}_{i,j}\end{aligned} \quad (6)$$

where $i = 1, 2, \cdots, N$, $j = 1, 2, \cdots, K$. Because the receive beamforming vector chosen to maximize the SINR is a minimum-mean-squared-error receiver, the received combining vector can be written as

$$\mathbf{w}_{i,j} = \left(\sum_{i}\sum_{j}\lambda_{i,j}\mathbf{H}_{i,j}^{*}\mathbf{H}_{i,j}+P_{i,j}\right)^{-1}\mathbf{H}_{i,j}/\sigma_{i,j} \quad (7)$$

The transmit beamforming vectors are chosen to satisfy the maximum ratio transmission criterion as

$$\mathbf{f}_{i,j} = \frac{\mathbf{w}_{i,j}\mathbf{H}_{i,j}}{\left\|\mathbf{w}_{i,j}\mathbf{H}_{i,j}\right\|},\ i = 1,\ldots,N,\quad j = 1,\ldots,K. \quad (8)$$

By substituting the original JOP problem with the newly obtained $\mathbf{w}_{i,j}$ and $\mathbf{f}_{i,j}$, the downlink minimization problem can be expressed as an uplink minimization problem as follows:

$$\text{maximize } \sum_i \sum_j \lambda_{i,j}\sigma_{i,j}^2$$
$$\text{subject to } \Lambda_{i,j} \geq \gamma_{i,j} \tag{9}$$
$$\text{where } \Lambda_{i,j} = \max_{\mathbf{w}_{i,j}, P_{i,j}} \frac{\lambda_{i,j}\left|\mathbf{w}_{i,j}^* \mathbf{H}_{i,k}\mathbf{f}_{i,k}\right|^2}{\sum_i \sum_j P_{i,j}\left|\mathbf{w}_{i,j}^* \mathbf{H}_{i,j}\mathbf{f}_{i,j}\right|^2 + P_{i,j}}$$

where $i = 1, \cdots, N$, $j = 1, \cdots, K$.

The Lagrangian dual JOP method has several advantages over other methods. First, the computation of the Lagrangian is simple because $\Lambda_{i,j}$ is only a function of $\mathbf{w}_{i,j}$ and $P_{i,j}$. This is a simple IOP in the uplink, which is equivalent to a JOP in the downlink. Second, the noise variance $\sigma_{i,j}^2$ is usually modeled as a constant, regardless of the user. Thus, we can further simplify the minimization solely over the Lagrangian $\lambda_{i,j}$ in (9).

Ⅳ. Numerical Results

The simulation parameters used are the same as those in [1]. We assumed 19 cell wrap-around scenarios with three sectors per cell to evaluate the intercell interference in a homogeneous manner. We used the Cost-Hata model as the propagation model, which is known to be a good path loss model with a carrier frequency of over 2 GHz. This is also efficient for the proposed algorithm because the number of main interferers in such a path loss model does not exceed three. Then, we simplified the beamforming vector selection process. The maximum transmit power per cell was set to 40 dBm, which is equivalent to 10 W, and the cell radius was 1000 m, which is very close to those of current networks. The generation of the channel matrices was completely random but followed a Gaussian distribution according to the central limit theorem as the number of iterations increased. Because the number of unknown parameters to be determined was at least NK, exact solution procedures or bounding and approximation schemes using Jensen's inequality or the Edmundson-Madansky inequality were not numerically possible unless there was a significantly lower number of N and K. Despite the existence of an optimality gap when using a Monte Carlo simulation, more than 1000 iterations were conducted to obtain a more reliable solution.

First, the similarity of the distribution patterns of the SNR and SINR distributions of the proposed algorithm and I-JOP was experimentally confirmed. As the two algorithms yielded the same optimal solution owing to their feasibility and convexity, they were indistinguishable in the simulation results. As shown in Table 1, although the proposed algorithm has much lower complexity than I-JOP, both algorithms performed similarly in terms of the total and per-user capacities.

Table 1. Performance comparison of the IOP, I-JOP, and proposed algorithms

	Total capacity	Capacity per user
IOP	135.0575 bit/sec/Hz	2.3694 bit/sec/Hz
I-JOP [1]	183.1052 bit/sec/Hz	3.1345 bit/sec/Hz
Proposed	183.7253 bit/sec/Hz	3.1571 bit/sec/Hz

Ⅴ. Conclusion

In this study, we proposed a joint beamforming and power optimization scheme using Lagrangian duality for cooperative MIMO Systems. The uplink problem (i.e., the Lagrangian dual JOP) was considerably easier to solve than the downlink problem because it was an individual optimization problem. Consequently, the proposed method had very low complexity compared to the existing iterative optimization method, I-JOP. In addition, although the SNR distribution of the proposed algorithm appeared worse than that of IOP, the SINR performance wa much better. Finally, we confirmed that the capacity performance of the proposed scheme was almost the same as that of I-JOP.

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